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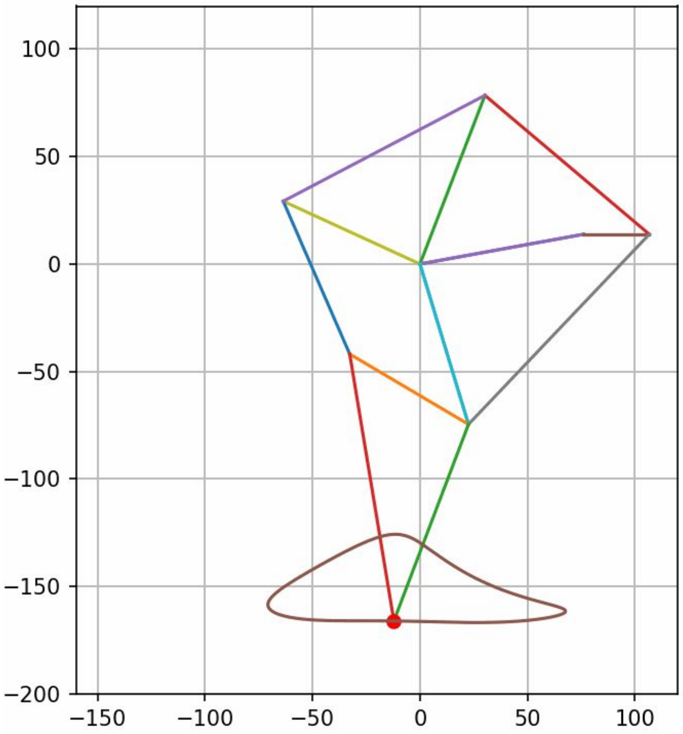
Design, Synthesis and Analysis of The Jansen Linkage

Introduction Jansen Linkage

- Planar leg mechanism
- Consists of 8 links
    - 1 Ternary
    - 7 Binary (one fixed)
- Mobility = 3(8-1)-2(10) = 1
- 1 DOF mechanism

Objective: Perform synthesis for 4 position motion generation with prescribed timing for the coupler (foot or the toe)

- Dimension synthesis
- Kinematic Analysis
- Dynamic Analysis
- Simulation
- Iterate



Dimension Synthesis

Formulation

Four Position Motion generation with Prescribed Timing (j=2,3,4)

$$z_5(e^{i\phi_{5j}}-1)+z_{62}(e^{i\phi_{6j}}-1)=\delta_j \qquad (1)$$

$$z_4(e^{i\phi_{4j}}-1)+z_{63}(e^{i\phi_{6j}}-1)=\delta_j+z_{33}(e^{i\phi_{3j}}-1) \qquad (2)$$

$$-z_{33}+z_4+z_{63}=z_5+z_{62} \qquad (3)$$

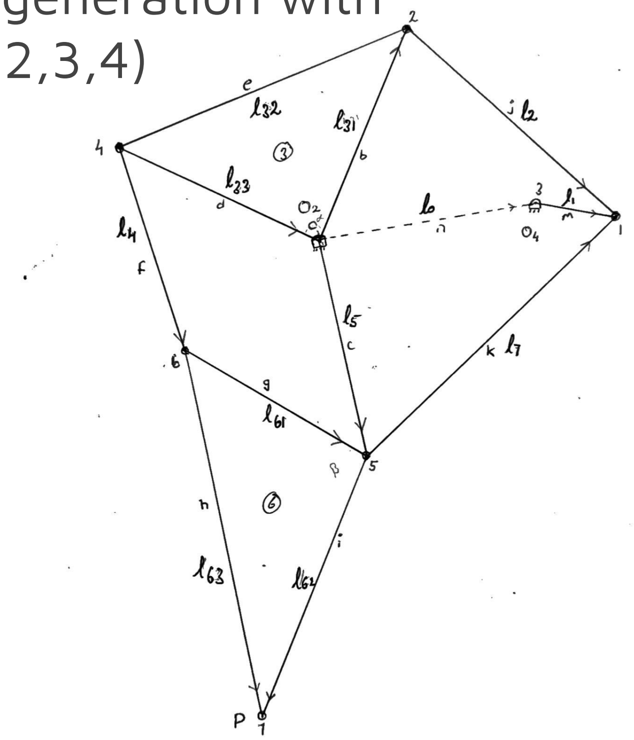
$$z_{31}(e^{i\phi_{3j}}-1)+z_2(e^{i\phi_{2j}}-1)=z_1(e^{i\phi_{1j}}-1) \qquad (4)$$

$$z_5(e^{i\phi_{5j}}-1)+z_7(e^{i\phi_{7j}}-1)=z_1(e^{i\phi_{1j}}-1) \qquad (5)$$

$$z_{31}+z_2=z_5+z_7 \qquad (6)$$

- **Total Scalar Equations**: $(8\times 3)+4=28$
- **Given**: $\delta_j, \phi_{1j}, \phi_{6j}$ 12 scalars

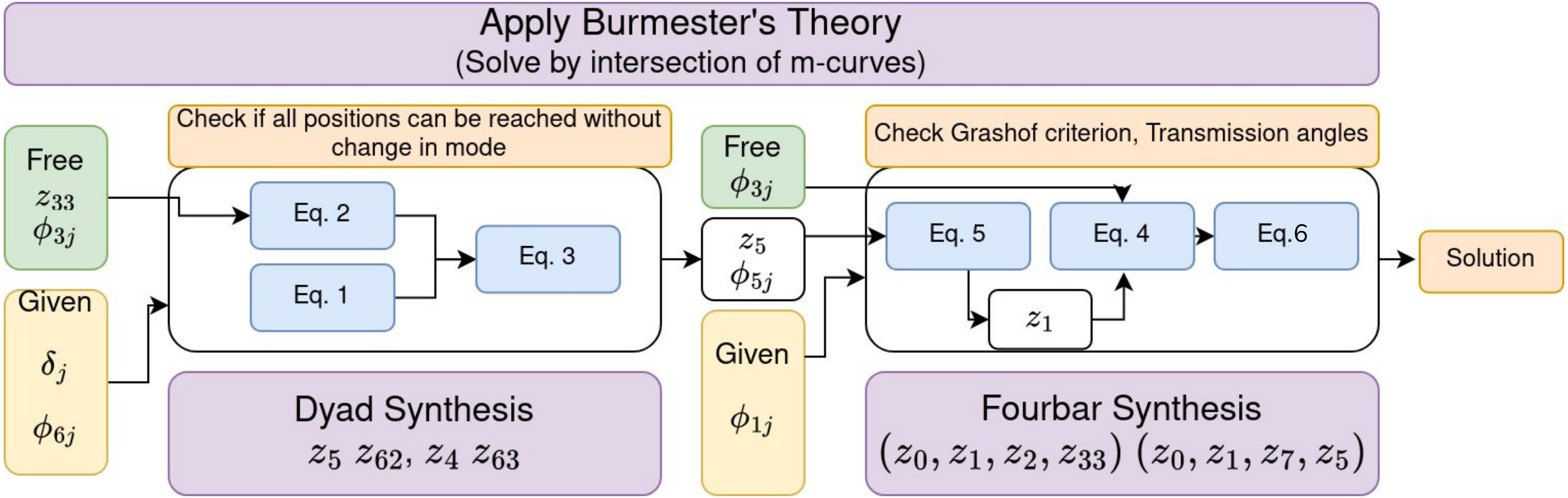
- **Total Unknowns**: 33 scalars
- z_5 z_{61} z_7 z_{63}
  z_1 z_{31} z_{33} z_2 z_4
  ϕ_{5j} ϕ_{3j} ϕ_{4j} ϕ_{2j} ϕ_{7j}
- **Free Choices**: z_{33} ϕ_{3j} $33-28=5$ scalars



Formulation Burmester Theory, Dyad Synthesis

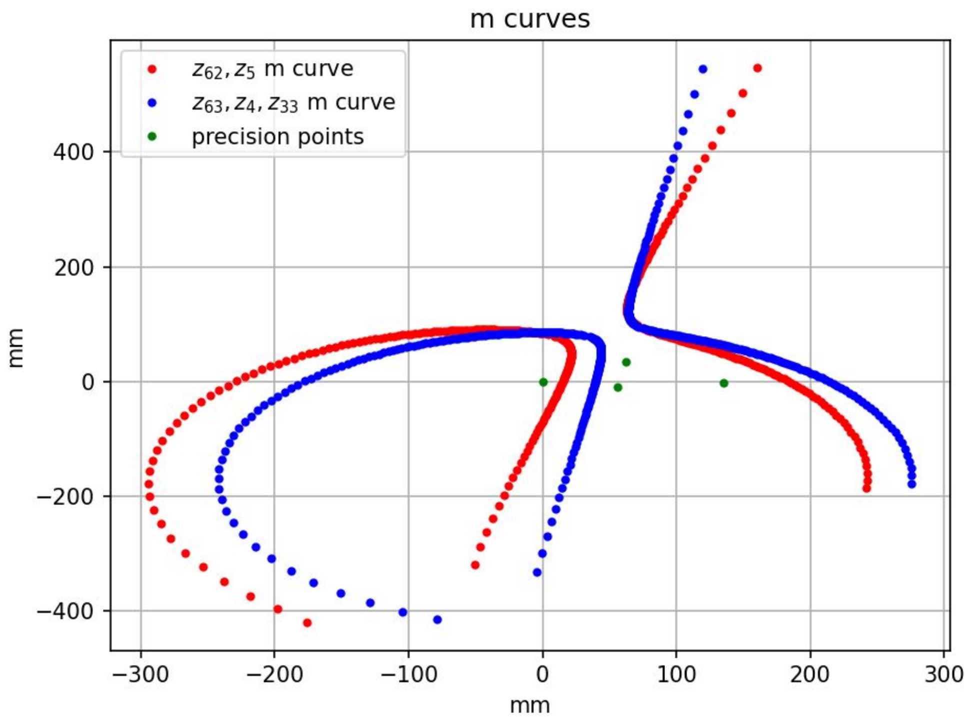
$$z_5(e^{i\phi_{5j}} - 1) + z_{62}(e^{i\phi_{6j}} - 1) = \delta_j \qquad (1)$$
$$z_4(e^{i\phi_{4j}} - 1) + z_{63}(e^{i\phi_{6j}} - 1) = \delta_j + z_{33}(e^{i\phi_{3j}} - 1) \qquad (2)$$
$$-z_{33} + z_4 + z_{63} = z_5 + z_{62} \qquad (3)$$

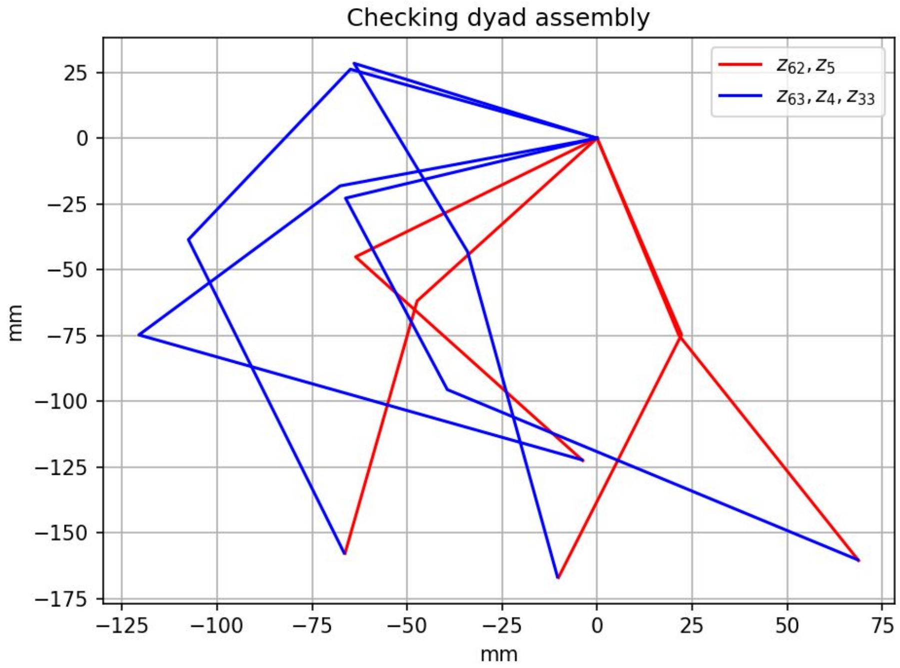
$$z_{31}(e^{i\phi_{3j}} - 1) + z_2(e^{i\phi_{2j}} - 1) = z_1(e^{i\phi_{1j}} - 1) \qquad (4)$$
$$z_5(e^{i\phi_{5j}} - 1) + z_7(e^{i\phi_{7j}} - 1) = z_1(e^{i\phi_{1j}} - 1) \qquad (5)$$
$$z_{31} + z_2 = z_5 + z_7 \qquad (6)$$



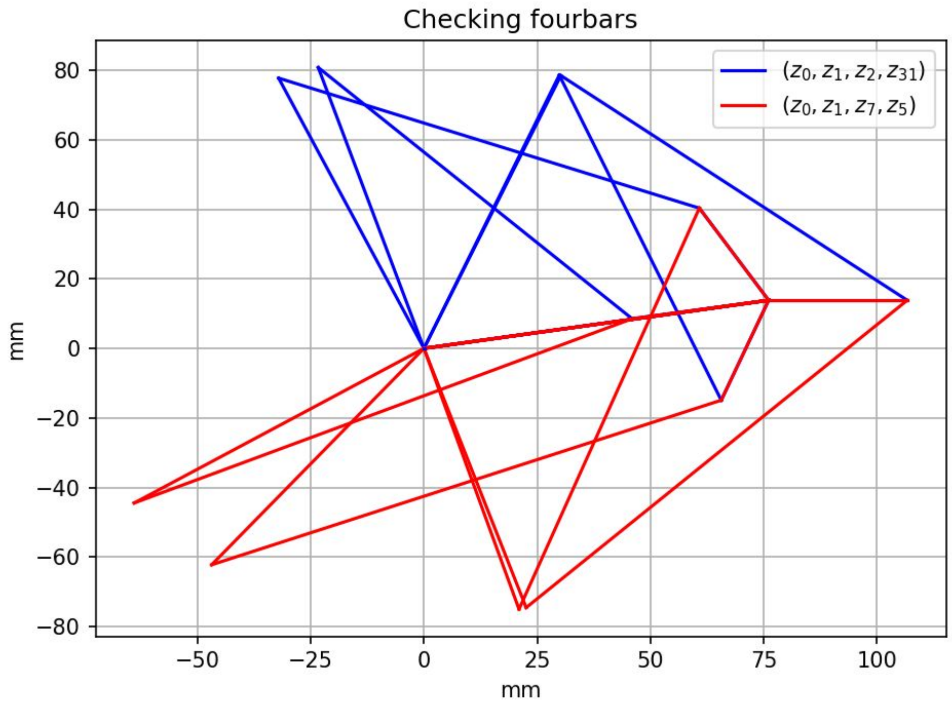
Dyad Synthesis - Finding Center points, Checking assembly

- All positions reached without change in mode





Four Bar Synthesis - Checking assembly, Grashof Criterion



- All positions reached without disassembly, in order
- Grashof criterion is satisfied for both fourbars, with z1 as crank in both cases

Finalized Design

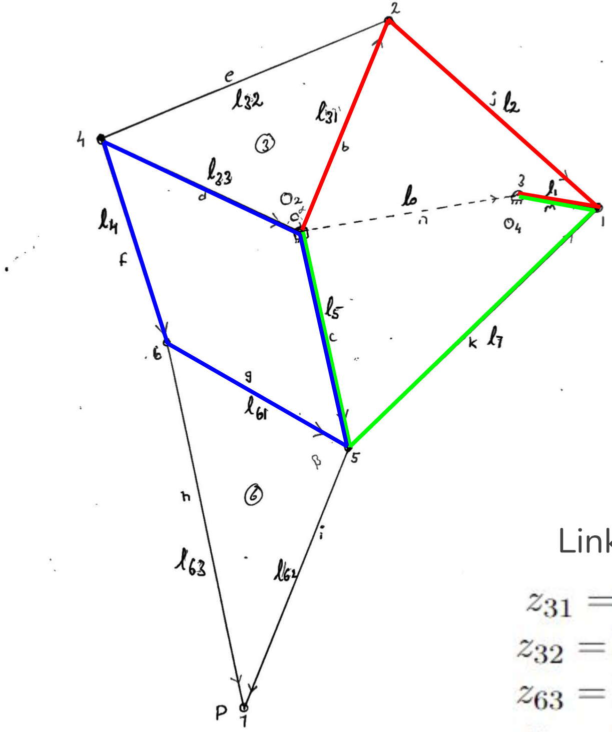
$\theta_0 = 0.1799\text{rad}$

$z_{31} = 84.088$	$z_5 = 77.909$	$z_{33} = 70.000$
$z_{32} = 105.978$	$z_4 = 77.535$	$z_{61} = 64.193$
$z_{63} = 125.849$	$z_{62} = 97.838$	$z_2 = 100.113$
$z_7 = 122.038$	$z_0 = 77.337$	$z_1 = 30.594$

Lengths in mm

Kinematic Analysis

Position Analysis Vector Loops



$$z_0 + z_1 = z_{31} + z_2 \qquad (1)\ \text{Vector Loop}$$

$$z_0 + z_1 = z_5 + z_7 \qquad (2)\ \text{Vector Loop}$$

$$z_{33} + z_5 = z_4 + z_{61} \qquad (3)\ \text{Vector Loop}$$

$$\theta_{33} = \theta_{31} + \alpha + \pi \qquad (4)\ \text{Rigid body}$$

$$\theta_{33} = \theta_{31} + \beta - \pi \qquad (5)\ \text{Rigid body}$$

$$P_7 = z_5 + z_{62} \qquad (6)\ \text{Vector Equation}$$

Link Lengths

$$\theta_0 = 0.1799\text{rad}$$

$z_{31} = 84.088$	$z_5 = 77.909$	$z_{33} = 70.000$
$z_{32} = 105.978$	$z_4 = 77.535$	$z_{61} = 64.193$
$z_{63} = 125.849$	$z_{62} = 97.838$	$z_2 = 100.113$
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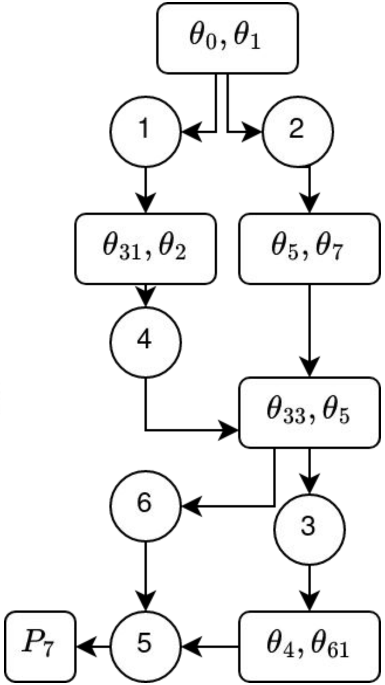


Velocity and Acceleration Analysis

Differentiating Loop Equations

For a four bar (a, b, c, d)

$$ae^{i\theta_a} + be^{i\theta_b} = ce^{i\theta_c} + de^{i\theta_d}$$ Position equation

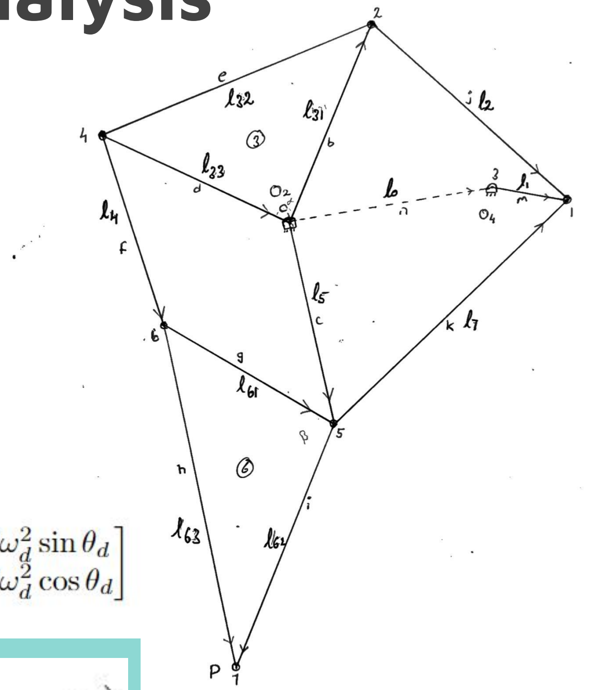
$$ae^{i\theta_a}\omega_a + be^{i\theta_b}\omega_b = ce^{i\theta_c}\omega_c + de^{i\theta_d}\omega_d$$ Velocity equation

$$\begin{bmatrix} c\cos\theta_c & d\cos\theta_d \\ c\sin\theta_c & d\sin\theta_d \end{bmatrix}\begin{bmatrix} \omega_c \\ \omega_d \end{bmatrix} = \begin{bmatrix} a\omega_a\cos\theta_a + b\omega_b\cos\theta_b \\ a\omega_a\sin\theta_a + b\omega_b\sin\theta_b \end{bmatrix}$$

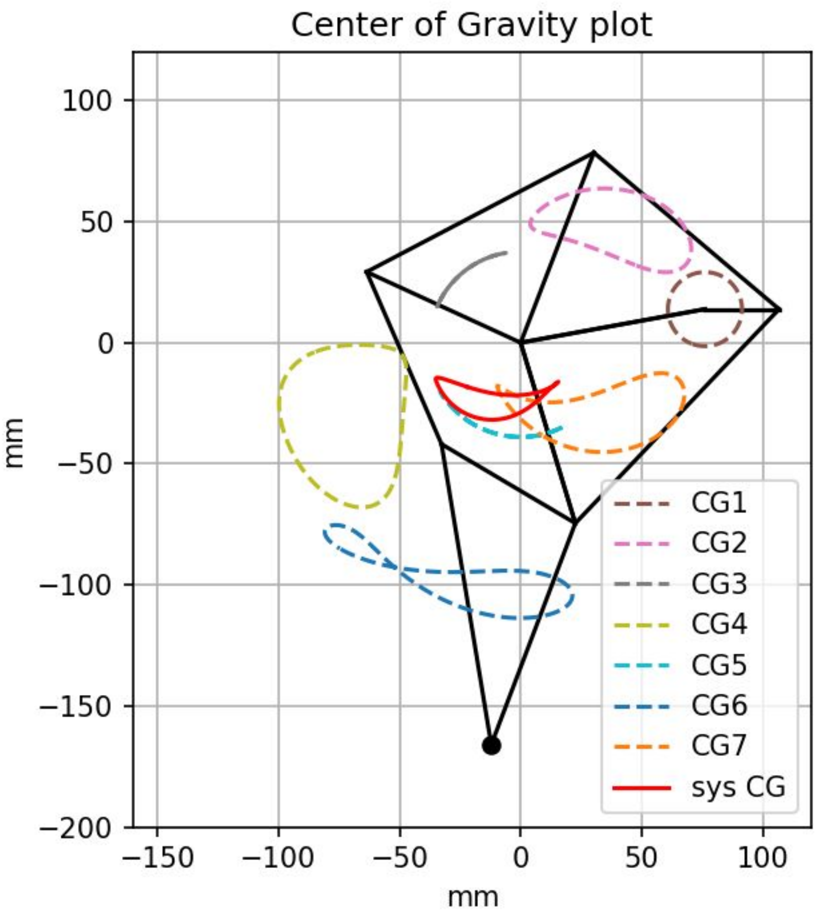
$$ae^{i\theta_a}(i\omega_a^2 + \alpha_a) + be^{i\theta_b}(i\omega_b^2 + \alpha_b) = ce^{i\theta_c}(i\omega_c^2 + \alpha_c) + de^{i\theta_d}(i\omega_d^2 + \alpha_d)$$ Acceleration equation

$$\begin{bmatrix} c\cos\theta_c & d\cos\theta_d \\ c\sin\theta_c & d\sin\theta_d \end{bmatrix}\begin{bmatrix} \omega_c \\ \omega_d \end{bmatrix} = \begin{bmatrix} a\alpha_a\cos\theta_a + b\alpha_b\cos\theta_b - a\omega_a^2\sin\theta_a - b\omega_b^2\sin\theta_b + c\omega_c^2\sin\theta_c + d\omega_d^2\sin\theta_d \\ a\alpha_a\sin\theta_a + b\alpha_b\sin\theta_b + a\omega_a^2\cos\theta_a + b\omega_b^2\cos\theta_b - c\omega_c^2\cos\theta_c - d\omega_d^2\cos\theta_d \end{bmatrix}$$

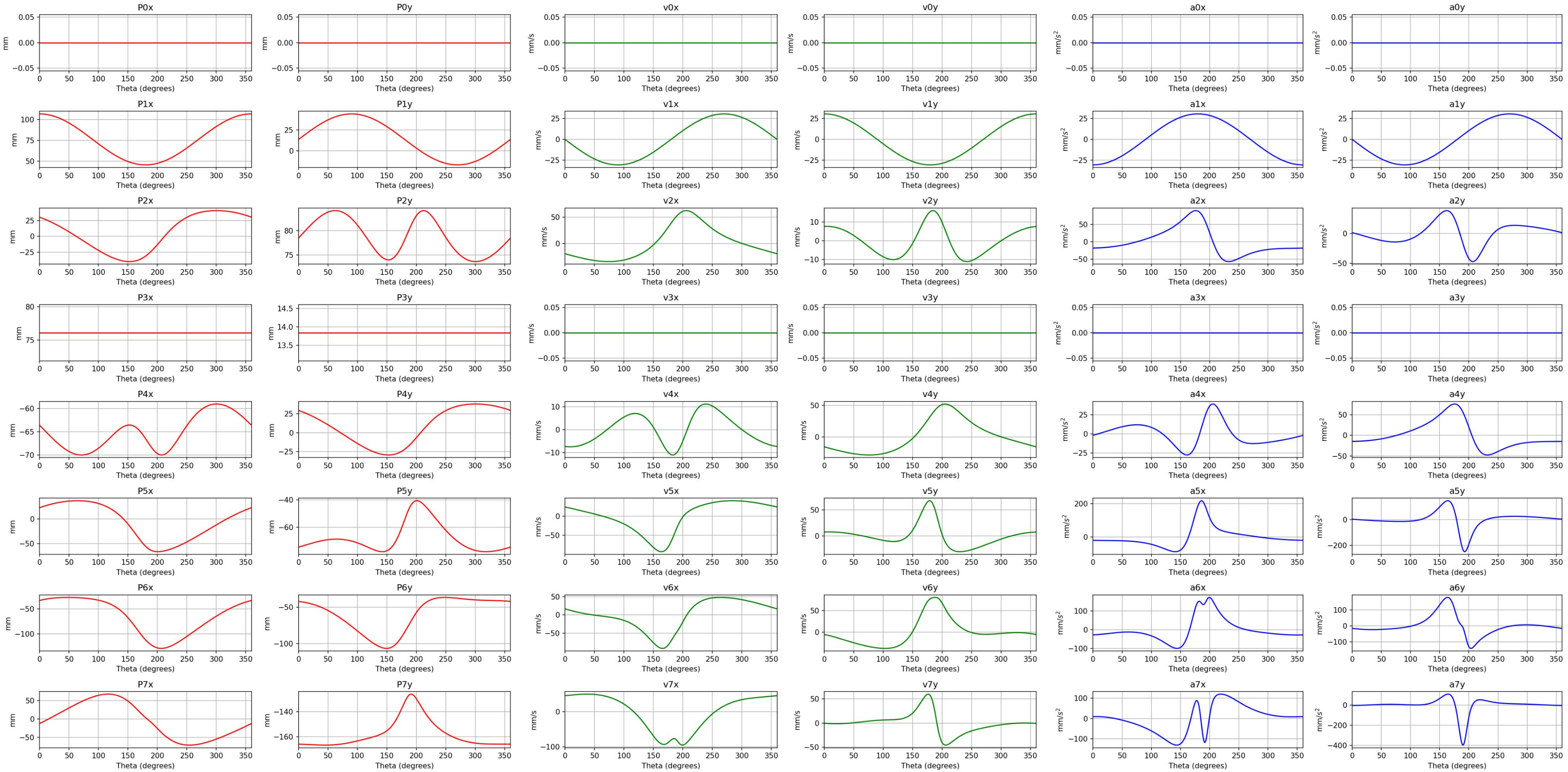
$$\boxed{(a, b, c, d) \equiv (z_0, z_1, z_2, z_{31}) \equiv (z_0, z_1, z_7, z_5) \equiv (z_{33}, z_5, z_{61}, z_4)}$$



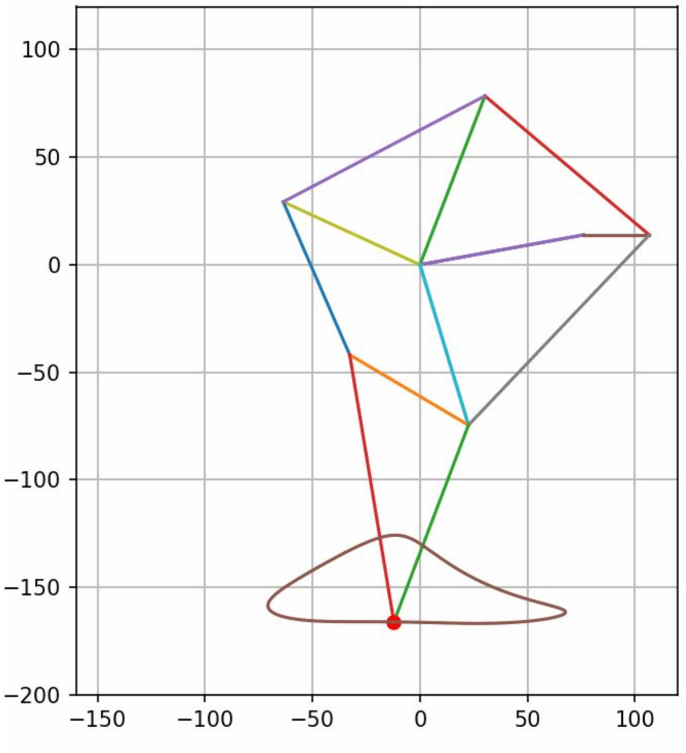
Motion of Center of Gravity of Links and the entire Linkage



Joint Position, Velocity and Acceleration plots (Input 1 rad/s)



Results



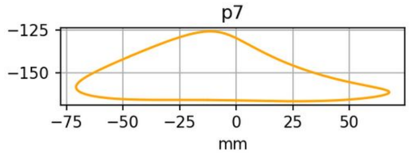
Requirements:

1. Grashoff Criterion: Verified for both 4 bar linkages:
   For $z_0z_1z_5z_7$: $z_1+z_7=30.594+122.038=152.632$ mm
   $z_5+z_0=77.909+77.337==155.246$ mm

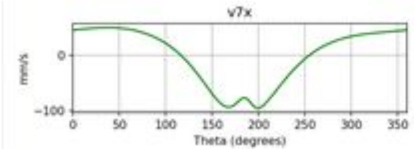
   For $z_0z_1z_2z_{31}$: $z_1+z_2= 100.113+30.594=130.707$ mm
   $z_0+z_{31}=84.088+77.337=161.425$ mm
2. Transmission Angles:

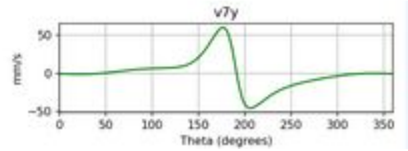
   For $z_0z_1z_5z_7$(Required>20): Min: 25° Max: 71°
   For $z_0z_1z_2z_{31}$: Min: 8° Max: 61°

3. Stride Length & Height:
   Length: 100mm
   Height: >120mm



4. $V_y = 0$ for when foot is in contact with ground:





Dynamic Analysis

Formulation

- 7 Moving Links -> 21 Equations

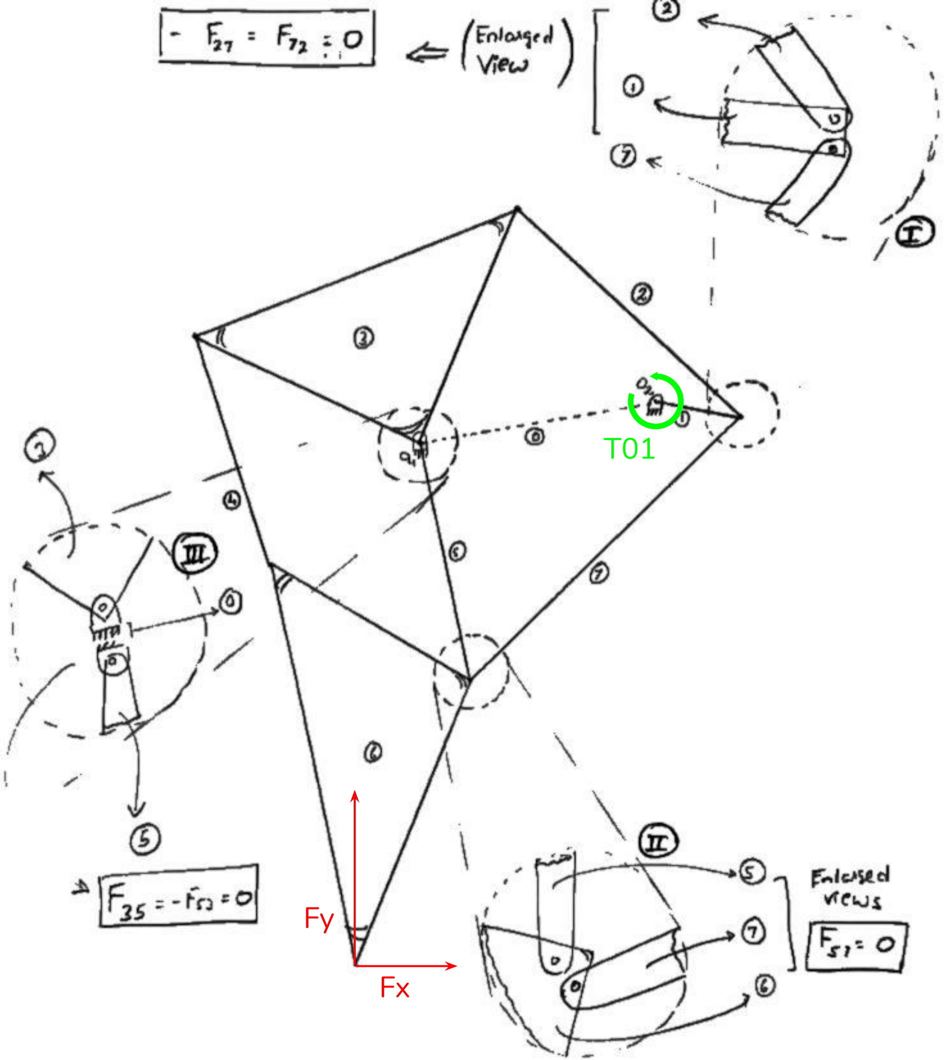
- Unknowns (27 scalar unknowns)
- Indeterminate

$$
\begin{array}{ccc}
F_{01} & F_{03} & F_{05} \\
F_{12} & F_{17} & F_{23} \\
F_{27} & F_{34} & F_{35} \\
F_{46} & F_{56} & F_{57} \\
F_{67} & T_{01} &
\end{array}
$$

- Convert all hinges to simple hinges
- 27-6 = 21 Unknowns

$$F_{27} = F_{35} = F_{57} = 0$$

- 21 Unknowns (Determinate)

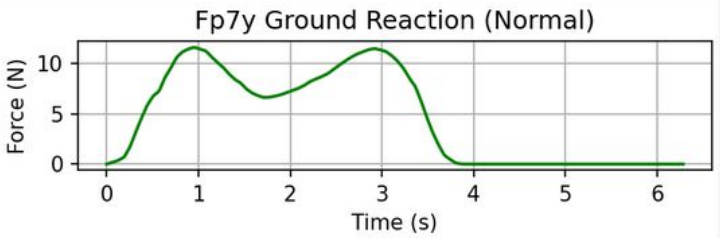


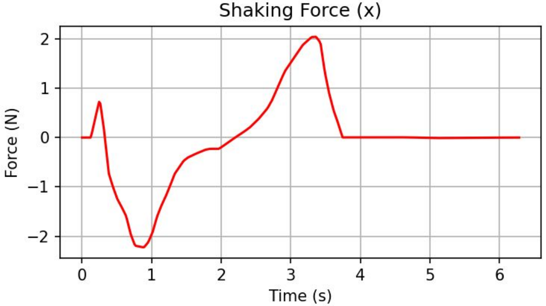
Formulation Newton-Euler

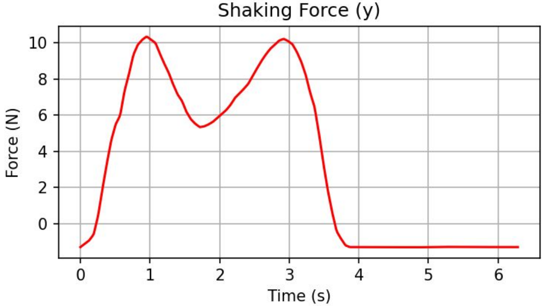
$$
\begin{bmatrix}
1 & 0 & 0 & -1 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 1 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 & 0 & 1 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 1 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & -1 & -1 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & -1 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 1 & -1 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & -1 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & -1 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 1 & 0 \\
-C_1P_{3y} & 0 & 0 & C_1P_{1y} & C_1P_{1y} & 0 & 0 & 0 & 0 & 0 & C_1P_{3x} & 0 & 0 & -C_1P_{1x} & -C_1P_{1x} & 0 & 0 & 0 & 0 & 0 & 1 \\
0 & 0 & 0 & -C_2P_{1y} & 0 & C_2P_{2y} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & C_2P_{1x} & 0 & -C_2P_{2x} & 0 & 0 & 0 & 0 & 0 \\
0 & -C_3P_{0y} & 0 & 0 & 0 & -C_3P_{2y} & C_3P_{4y} & 0 & 0 & 0 & 0 & C_3P_{0x} & 0 & 0 & 0 & C_3P_{2x} & -C_3P_{4x} & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & -C_4P_{4y} & C_4P_{6y} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & C_4P_{4x} & -C_4P_{6x} & 0 & 0 & 0 \\
0 & 0 & -C_5P_{0y} & 0 & 0 & 0 & 0 & 0 & C_5P_{5y} & 0 & 0 & 0 & C_5P_{0x} & 0 & 0 & 0 & 0 & 0 & -C_5P_{5x} & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & -C_6P_{6y} & -C_6P_{5y} & C_6P_{5y} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & C_6P_{6x} & C_6P_{5x} & -C_6P_{5x} & 0 \\
0 & 0 & 0 & 0 & -C_7P_{1y} & 0 & 0 & 0 & 0 & -C_7P_{5y} & 0 & 0 & 0 & 0 & C_7P_{1x} & 0 & 0 & 0 & 0 & C_7P_{5x} & 0
\end{bmatrix}
\begin{bmatrix}
F_{01x} \\ F_{03x} \\ F_{05x} \\ F_{12x} \\ F_{17x} \\ F_{23x} \\ F_{34x} \\ F_{46x} \\ F_{56x} \\ F_{67x} \\ F_{01y} \\ F_{03y} \\ F_{05y} \\ F_{12y} \\ F_{17y} \\ F_{23y} \\ F_{34y} \\ F_{46y} \\ F_{56y} \\ F_{67y} \\ T_{01}
\end{bmatrix}
=
\begin{bmatrix}
m_1a_{1x} \\ m_2a_{2x} \\ m_3a_{3x} \\ m_4a_{4x} \\ m_5a_{5x} \\ m_6a_{6x}-F_x \\ m_7a_{7x} \\ m_1(a_{1y}+g) \\ m_2(a_{2y}+g) \\ m_3(a_{3y}+g) \\ m_4(a_{4y}+g) \\ m_5(a_{5y}+g) \\ m_6(a_{6y}+g)-F_y \\ m_7(a_{7y}+g) \\ I_1\alpha_1 \\ I_2\alpha_2 \\ I_3\alpha_3 \\ I_4\alpha_4 \\ I_5\alpha_5 \\ I_6\alpha_6-(C_6P_{7x}F_y-C_6P_{7y}F_x) \\ I_7\alpha_7
\end{bmatrix}
$$

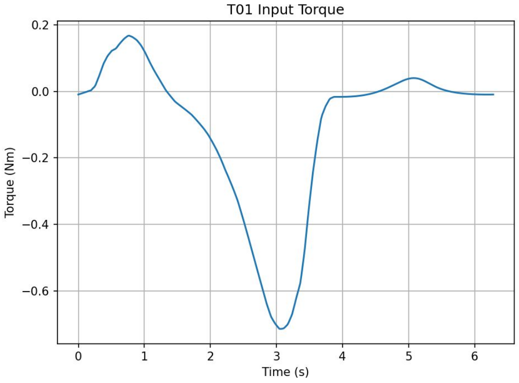
- Objective input link is to be maintained at 1 rad/s, to determine input torque
- Links are considered to have mass (gravity acts vertically downwards)
- Ground reaction forces (Fy, Fx) are modelled in 2 ways
    - By Gait analysis (Whittle's Gait Analysis)
    - As a step force (constant)

Joint Forces (Input 1 rad/s) (Foot subject to forces from gait analysis)









Observations:

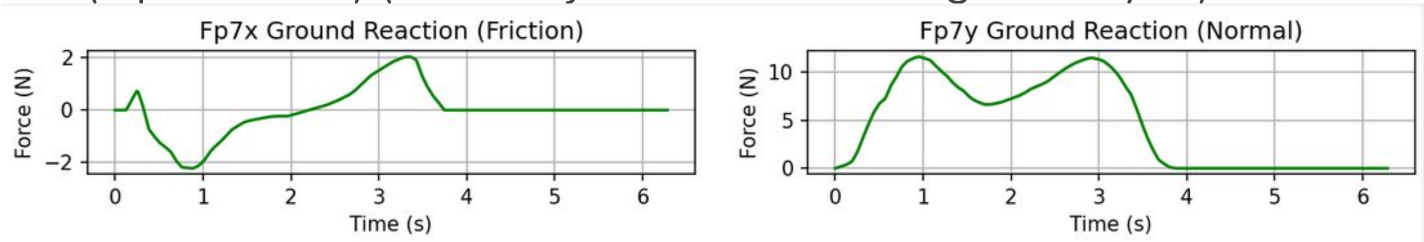
- The shaking forces are periodic, and vary within permissible limits of deformation:

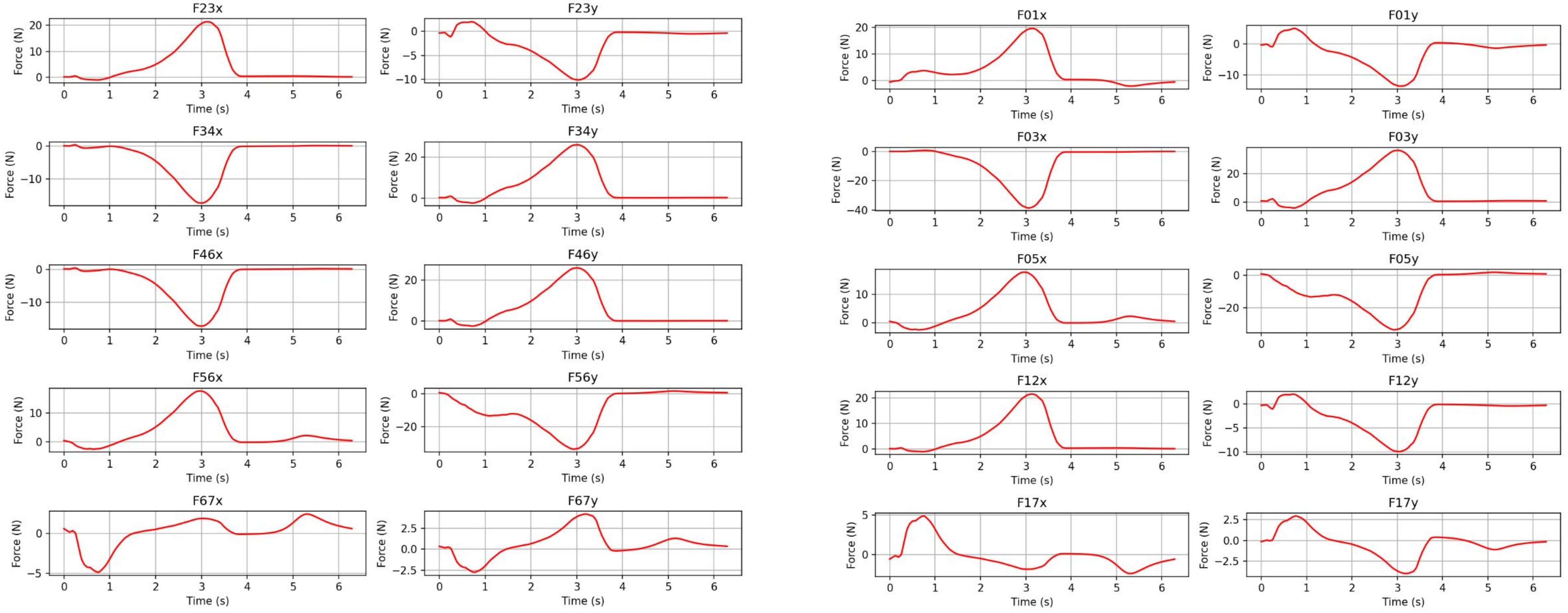
Shaking Forces	Min(N)	Max(N)
x	-2.25	2
y	-1.8	10

- The motor’s torque characteristics must be be capable of handling changes in torque as shown in the plot.

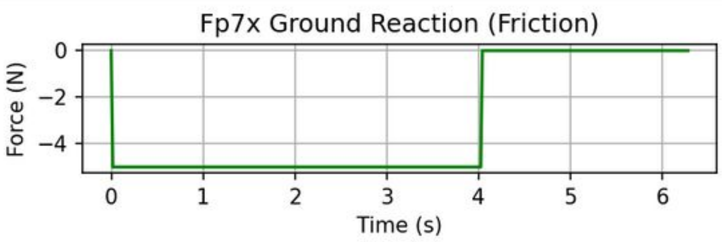
	Min(Nmm)	Max(Nmm)
Input Torque	-0.75	0.15

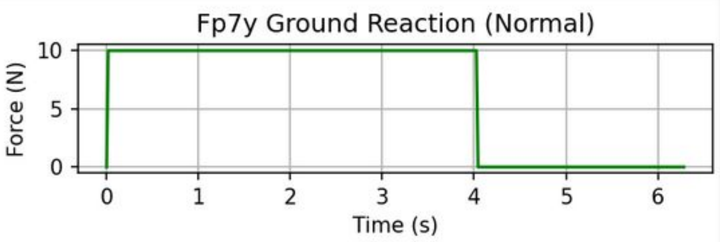
Joint Forces (Input 1 rad/s) (Foot subject to forces from gait analysis)

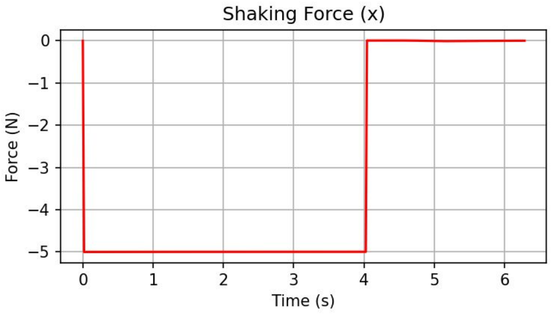


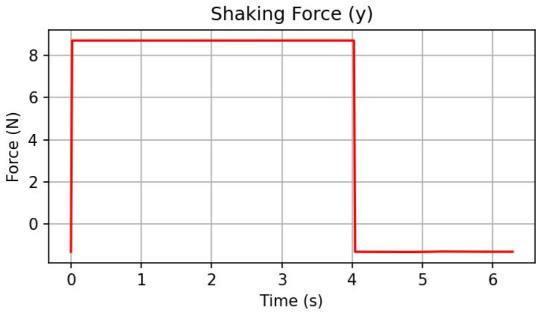


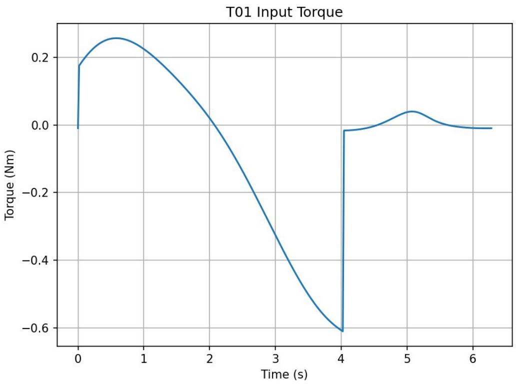
Joint Forces (Input 1 rad/s) (Foot Subject to Step Input)











Observations:

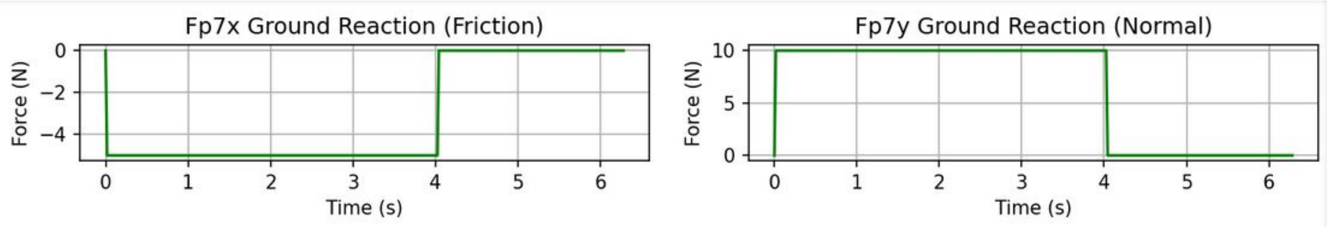
- The shaking forces are periodic, and vary within permissible limits of deformation:

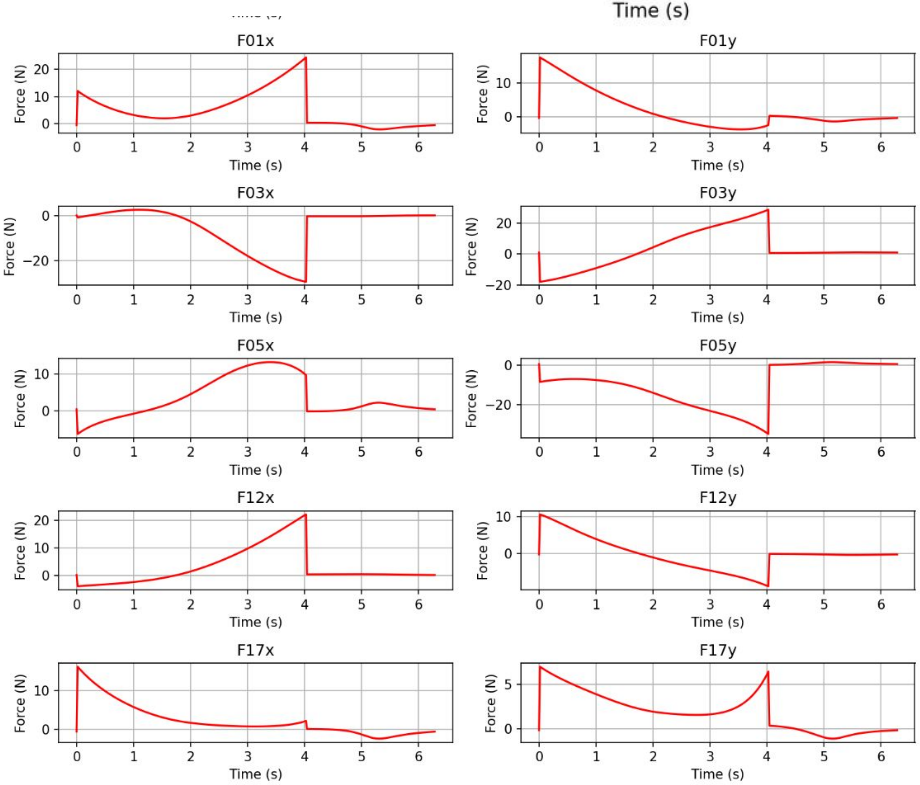
Shaking Forces	Min(N)	Max(N)
x	-5	0
y	0	10

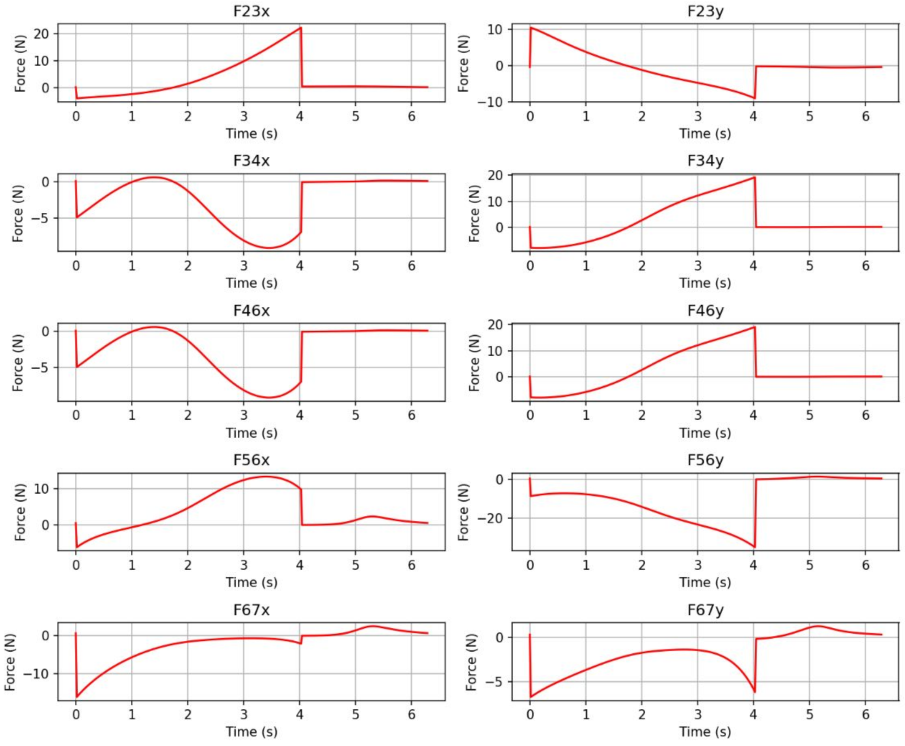
- The motor's torque characteristics must be be capable of handling changes in torque as shown in the plot:

	Min(Nmm)	Max(Nmm)
Input Torque	-0.6	0.25

Joint Forces (Input 1 rad/s) (Foot Subject to Step Input)

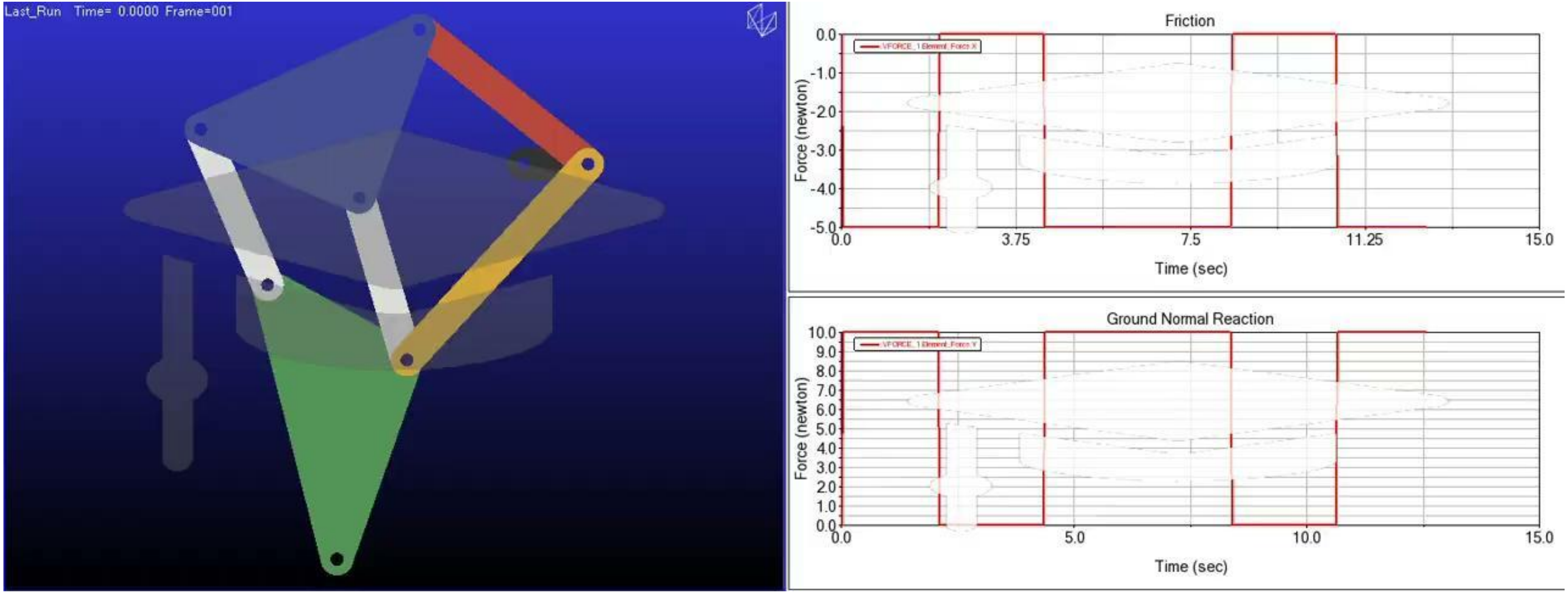






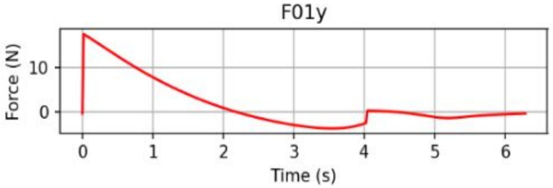
Simulation - ADAMS View

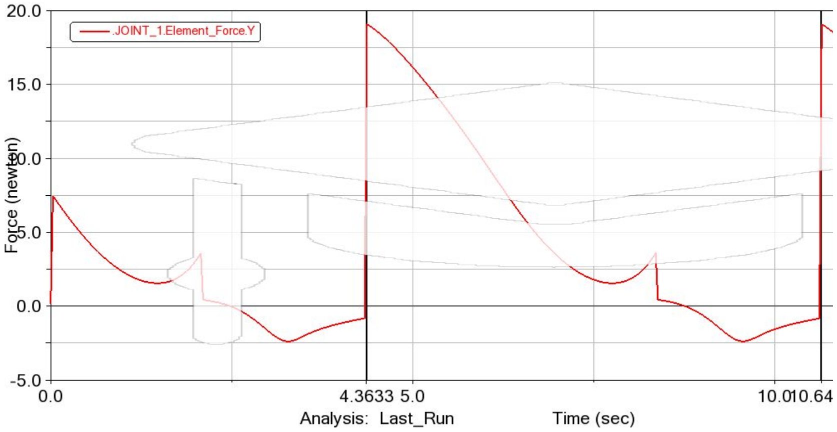
Simulation for Foot Subject to Step Input: (Input 1 rad/s)



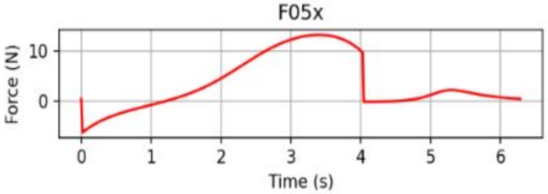
Verification of Joint Forces: Analytical vs Simulation

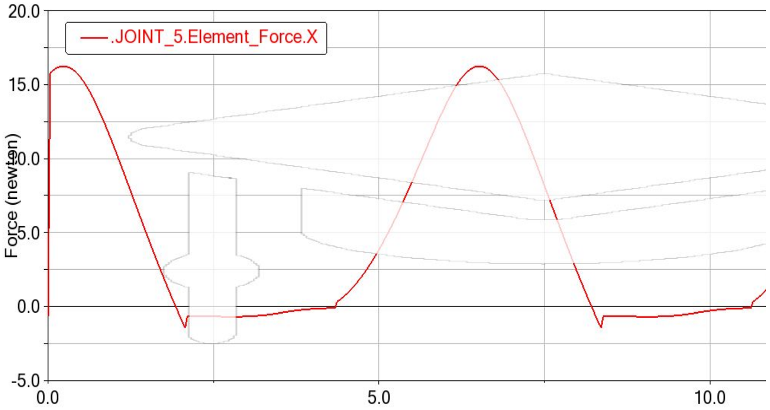
Ex1. F01

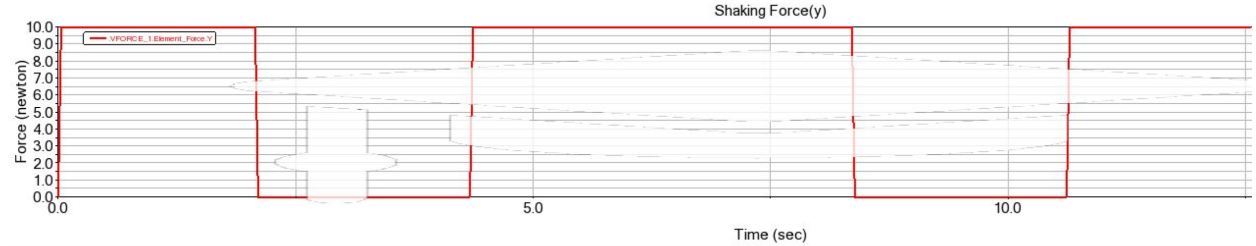


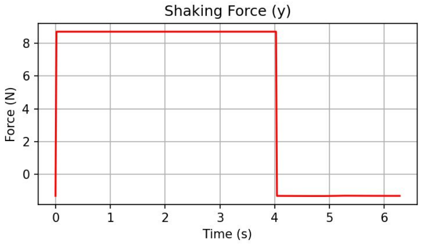


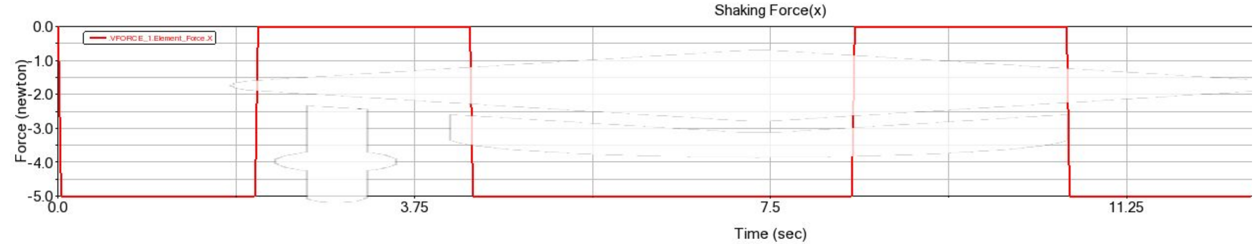
Ex1. F05

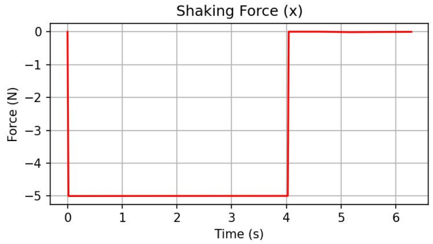


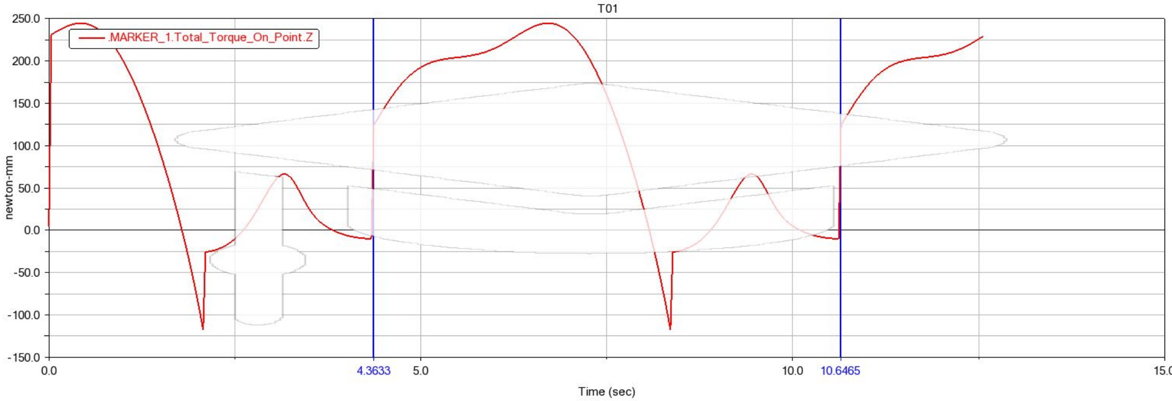


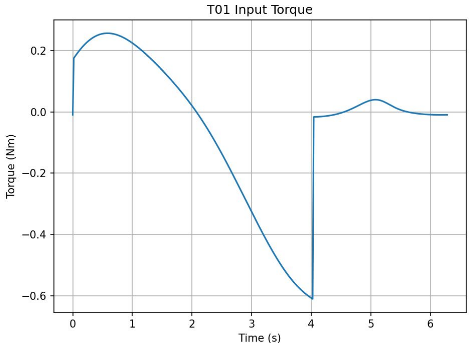












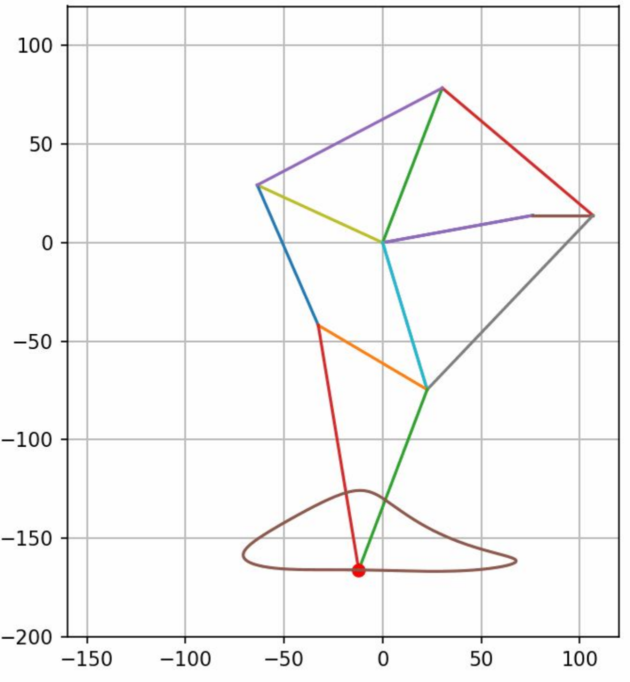
Verification of Shaking Forces & Input Torque Required: Simulation vs Analytical

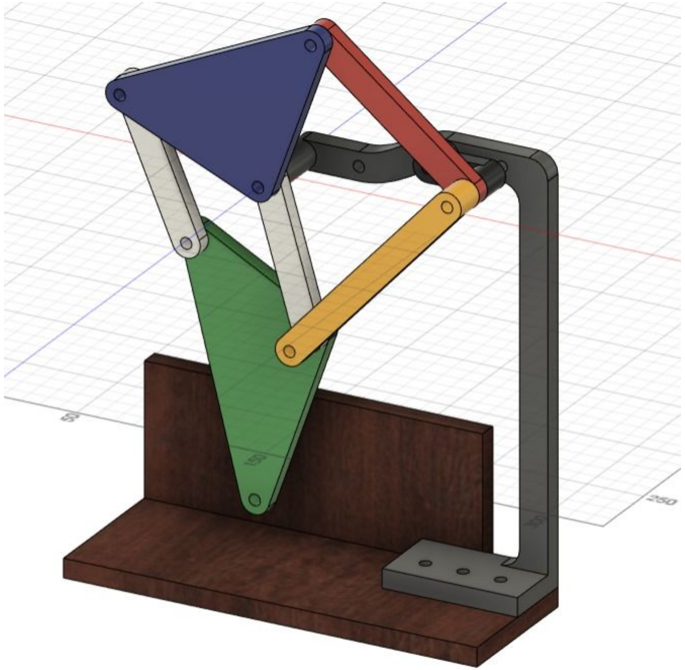
Manufacturing and Testing

Considerations while Designing

- Ensuring no interference between links by pushing links slightly out of plane
- Ensuring weight balance amongst planes
- Accounting for space taken by bolt heads and nuts, and space to tighten bolts
- Accounting for shrinkage during 3D printing- holes made larger than necessary

Design Process Results





Project Contributions

- Lalit Jayanti ME21B096
  - Dimension Synthesis: (Formulation, Programming, Plotting)
    - Applied Burmester theory and dyad synthesis techniques for synthesizing link lengths.
  - Kinematic Analysis (Position, Velocity, Acceleration): (Formulation, Programming, Plotting)
    - Wrote vector loop equations to find positions, differentiated the loop equations for velocity and acceleration.
  - Dynamic Analysis: (Formulation, Programming, Plotting)
    - Used the Newton-Euler technique to frame 21 equations of forces for the linkage to determine Torque requirement at the crank for 2 different force profiles at the foot or toe.

- Mugdha Meda ME21B119
  - Simulation and CAD Modeling: (ADAMS View)
    - Applied appropriate constraints for kinematic and dynamic analysis, analytical solutions verified.
    - Shaking forces, Torques required simulated for step input at foot.
  - Manufacturing of Functioning Prototype: (3D Print; Wood Work)
    - 3D printed designed links, cut wood for base, coupler boards, assembled with washers for smooth joint movement.

- Laasya Agarwaal
  - Report writing
  - Data analysis for Kinematics and Dynamics of the mechanism